Axion, Photons in terms of “Particles” and “Antiparticles”

E.Guendelman,
Ben Gurion U. , Israel,

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The Axion Photon System is described by the action

$$S = \int d^4x \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{g}{8} \phi \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \right].$$

Consider an external magnetic field pointing in the x direction

with magnitude B(y,z).

For small axion and photon perturbations which depend only on y, z and t, consider only up to quadratic terms in the perturbations.

Then the axion photon interaction is

$$S_I = -\int d^4x\,[\beta\phi E_x],$$

where $\beta = gB(y,z)$. Choosing the temporal gauge

- Considering also only x polarizations of the photon, since only this polarization couples to the axion and to the external magnetic field, we obtain that (A represents the x-component of the vector potential)

$$E_x = -\partial_t A$$

Ignoring integration over x (since everything is taken to be x-independent), we obtain the effective 2+1 dimensional action

$$S_2 = \int dy\, dz\, dt \left[\frac{1}{2} \partial_\mu A \partial^\mu A + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \qquad - \frac{1}{2} m^2 \phi^2 + \beta \phi \partial_t A \right]$$

Neglecting the mass of the axion, which gives O(2) symmetry in the kinetic term between photon and axion, performing an integration by parts in the interaction part of the action that gives the O(2) symmetric form for the interaction in the case the external magnetic field is static

$$S_I = \frac{1}{2}\int dy\,dz\,dt\,\beta\left[\phi\partial_t A - A\partial_t\phi\right]$$

In the infinitesimal limit there is an Axion Photon duality symmetry (0rdinary rotation in the axion photon space), here epsilon is an infinitesimal parameter

$$\delta A = \epsilon\phi , \qquad \delta\phi = -\epsilon A$$

Using Noether`s theorem, we get a conserved charge out of this, the charge density being given by

$$j_0 = A\partial_t\phi - \phi\partial_t A - \frac{\beta}{2}\left(A^2 + \phi^2\right)$$

Defining a complex scalar field

$$\psi = \frac{1}{\sqrt{2}}(\phi + iA),$$

We see the to first order in the external field the axion photon system interacts with the charge density which is like that of scalar electrodynamics

$$j_0 = i(\psi^* \partial_t \psi - \psi \partial_t \psi^*) - \beta \psi^* \psi$$

In the scalar QED language, the complex scalar creates particles with positive charge while the complex conjugate creates antiparticles with the opposite charge. The axion and photon fields create however linear contributions of states with opposite charges since

$$\phi = \frac{1}{\sqrt{2}}(\psi^* + \psi), \qquad A = \frac{1}{i\sqrt{2}}(\psi - \psi^*)$$

The Scalar QED Picture and its consequences

1. gB(y,z) couples to the “density of charge” like an external electric potential would do it.
2. The axion is a symmetric combination of particle antiparticle, while the photon is the antisymmetric combination.
3. If the direction of initial beam of photons or axions is perpendicular to the magnetic field and to the gradient of the magnetic field, we obtain in this case beam splitting (new result).
4. Known results for the cases where the direction of the beam is orthogonal to the magnetic field but parallel to the magnetic field gradient can be reproduced easily.

Known Results B=B(z),axion and photon fields functions of t, z

- This situation is not related to spitting, it is a problem in a potential with reflection and transmission. Here the particle and antiparticle components feel opposite potentials and therefore have different transmission coefficients t and T.
- Represent axion as (1,1) and photon as (1, -1).
- Then axion = (1,1) after scattering goes to (t,T).
- (t,T)=a(1,1)+b(1,-1), a=(t+T)/2, b=(t-T)/2= amplitude for an axion converting into a photon
- For initial photon=(1,-1) we scatter to (t, -T)=c(1,1)+

d(1,-1), so we find that c= b=(t-T)/2, d= a=(t+T)/2. Notice the symmetries: amplitude of axion going to photon =

amplitude of photon going to axion and amplitude for photon staying photon = amplitude for an axion staying an axion.

First order scattering amplitudes for a particle in an external electromagnetic field is (Bjorken&Drell)

$$S_{\mathbf{p}'_+\mathbf{p}_+} = -\frac{ie}{(2\pi)^3}\int \frac{d^4y}{\sqrt{2\omega_{p_+}2\omega_{p'_+}}} e^{iq\cdot y}(p_+ + p'_+)_\mu A^\mu(y) = \frac{-ie(p_+ + p'_+)_\mu A^\mu(q)}{(2\pi)^3\sqrt{2\omega_{p_+}2\omega_{p'_+}}}$$

$$q^\mu = p'^\mu_+ - p^\mu_+$$

where $A^\mu(q) = \int d^4y e^{iq_\nu \cdot y^\nu} A^\mu(y)$

In our case the analog of the e x (zeroth component of 4- vector potential) is gB(y,z), no spatial components of 4-vector potential exist

- x independence of our potential ensures conservation of x component of momenta (that is, this is a two spatial dimensions problem)
- t independence ensures conservation of energy
- the amplitude for antiparticle has opposite sign, is -S
- Therefore an axion, i.e. the symmetric combination of particle antiparticle (1,1) goes under scattering to (1,1) +(S, -S), S being the expression given before. So the amplitude for axion going into photon (1,-1) is S, this agrees with a known result obtained by P. Sikivie many years ago for this type of external static magnetic field.

The “Classical” CM Trajectory

- If we look at the center of a wave packet, it satisfies a classical behavior (Ehrenfest). In this case we get two types of classical particles that have + or – charges.
- In the presence of an inhomogeneuos magnetic field, these two different charges get segregated.
- This can take place thermodynamically or through scattering (to see this effect clearly one should use here wave packets, not plane waves!).

Thermodynamic Splitting

- In the classical limit the particles have a kinetic energy and a potential energy gB
- The antiparticles have the same kinetic energy but a potential energy –gB
- The ratio of particles to antiparticle densities at a given point is given by the corresponding ratios of Boltzmann factors, that is $\exp(-2gB(y,z)/kT)$.

Splitting through scattering

- From the expression of photon and axion in terms of particle and anti particle, we see that in the “classical” limit these two components move in different directions.
- If the direction of the initial beam is for example orthogonal to both the magnetic field and the direction of the gradient of the magnetic field, we obtain splitting of the particle and anti particle components
- There appears to be a radical difference between the case where spitting takes place, as opposed to the “frontal” case: in the splitting case, because the final momenta are different, the relative phases of particle and antiparticle grow even after we come out of interaction region.

The Extreme Far Region

- In fact if we take the particle antiparticle splitting picture seriously, and consider even a very small splitting angle, in any case we can take the Extreme Far Region,
- In this limit the particle and antiparticle components will be separated, each of these components is 50% axion, 50% photon, so by going very far we get an effect of order 1!.

Estimates

- Beam splitting, Konstantin Zioutas taught me: take distance between the beams of order de Broglie wave length, then for a magnetic field gradient of 1Tesla/cm, acting 10cm in the direction orthogonal to beam, we get splitting at L=1000,000km, for g close to upper bound.
- 1/L , -1/L are the momenta aquired
- Splitting represents O(1) effect, to much to ask, so what is obtained for smaller distances?. Here we will use models,

Rough estimate of amplitudes, using a plane wave model!

- The particle an antiparticle suffer a phase difference which increases with distance, even when we go out of interaction region, since they have aquired different momenta in the y direction: in natural units increment 1/L for particle, -1/L for antiparticle. So axion, represented by (1,1) becomes (exp(iy/L), exp(-iy/L)) = a(1,1)+b(1,-1). Which can be solved for b giving b= i sin(y/L). For y/L<<1, we get that amplitude of axion going into photon is iy/L.

- For y=L=1000,000km, probability is of order 1, in agreement with criterion for splitting. For y=10mt, we get probabilities of the order of more well known experiments. For y>10mt we would be doing better.

Towards more realistic estimates

- In the splitting effect one parameter that has to be considered is the width of the wave packet, how do we know that for axions coming from the sun?. Obviously for smaller widths it is easier to separate the particle and antiparticle packets (initially overlaping).
- Let us do then next rough model: Suppose we have axion, represented as two wave packets of particle antiparticle of width d(t). They suffer scatterings obtaining momenta 1/L and -1/L, which we calculated before (L=1000,000km) in the y direction. The two beams separate as (1/LE)t = (1/LE)z (z being direction of propagation of initial beam and we use c=1 units), as z>LEd, we get separation of particle and antiparticle .

Take for example d=const. and

- That the amplitude of photons produced will be linear in z.
- At z=LEd, we get O(1) effect (50% conversion).
- This means amplitude of photons approximately (z/LEd). Prob. = Square of that.

Conclusions

- Axion Photon interactions with an external magnetic field can be understood in terms of scalar QED notions.
- Standard, well known results can be reproduced, which gives this approach some pedagogical appeal (I teach a lot!).
- Photon and Axion splitting in an external inhomogeneous magnetic field is obtained.
- By observing at large distances from interaction region, effect can be amplified. Several estimates discussed. Work with P. Sikivie and K.Zioutas and Baker should clarify these points.