New theoretical ideas: Anomaly induced effects in magnetic field and at LHC

Oleg Ruchayskiy

École Politechnique Fédérale de Lausanne

Alexey Boyarsky

ETH Zürich & CERN

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Axion and axion-like particles

- In general, axion-like particle (**ALP**) is a pseudoscalar

$$\mathcal{L}_{\text{ALP}} = \frac{1}{2}(\partial_\mu a)^2 - \frac{m_a^2}{2}a^2 + \frac{a}{4M}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$$

- ALP couples to electromagnetic field via

$$\frac{1}{4}a(x)\,\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho} = a(x)\vec{E}\cdot\vec{H}$$

.

- One can search for ALPs in parallel electric and magnetic fields
- Other theories with $\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$?

Term $F\tilde{F}$

- $\tilde{F}F$ – term is total derivative

$$\tilde{F}F = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho} = \partial_\mu K^\mu$$

- or for non-Abelian fields

$$\tilde{F}F = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}\,\mathrm{Tr}\Big(F_{\mu\nu}F_{\lambda\rho}\Big)$$

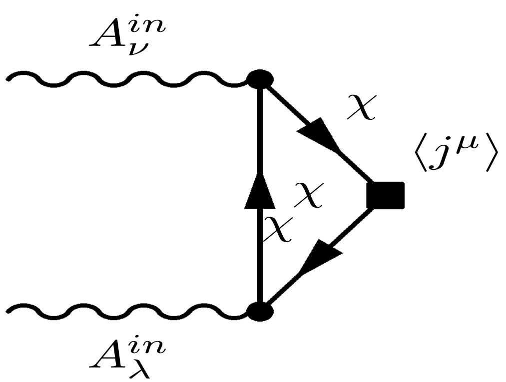
- (Chern-Simons) current K^μ is **not gauge invariant**
- Term $\int d^4x\,\tilde{F}F \neq 0$ is topological (does not depend on the metric)
- Related to **quantum anomalies**
- . . . and to "index theorem"

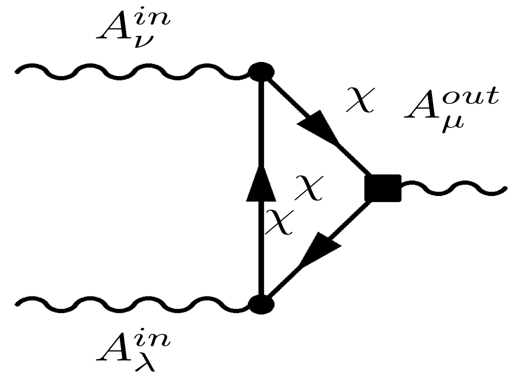
Anomalies

- Maxwell equations need conserved current:

$$\partial_\mu F^{\mu\nu} = j^\nu \quad \Rightarrow \quad \partial_\nu \partial_\mu F^{\mu\nu} = \partial_\nu j^\nu = 0$$

- If matter is quantum, the expectation value $\langle \partial_\mu j^\mu \rangle$ **= 0**
- Normally it is guaranteed by gauge symmetry.
- Loops of **chiral fermions** χ violate symmetries of classical theory:





$$k^\mu A_\mu^{\text{out}} \neq 0$$

$$\partial_\mu \langle j^\mu \rangle = \frac{e^3}{16\pi^2} \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho}$$

For a gauge symmetry this leads to non-unitarity.

Fermions in the Standard Model are chiral

How such a theory can be consistent?

Several chiral fermions can help make theory well defined

Fermions in the Standard Model

- Fermions of the Standard Model are chiral with respect to the $SU(2) \times U_Y(1)$ group.

$$\underbrace{L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad e_R,\ \nu_R(?)}_{\text{leptons}} \qquad \underbrace{Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad u_R,\ d_R}_{\text{quarks}}$$

- How to write masses for such fermions?
- Mass term mixes left and right-moving fermionic modes:

$$\mathcal{L}_{\text{mass}} = M\bar{\psi}\psi = M(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

- In chiral theories $e_L \neq e_R$. Mass term is **not gauge invariant**

Masses of fermions in the Standard Model

- Chiral fermions can obtain mass only through Yukawa interaction with the charged scalar (Higgs field): $H = H_1 + iH_2$

$$\mathcal{L}_{\text{Yukawa}} = f\bar{\psi}(H_1 + i\gamma_5 H_2)\psi = (fv)(\bar{\psi}_L e^{-i\theta}\psi_R + \bar{\psi}_R e^{i\theta}\psi_L)$$

mass of the fermion, $v = |H|$, θ – phase of the complex scalar field.

- Yukawa terms with the $SU(2)$ Higgs double H ($\tilde{H}^a = \epsilon^{ab}H_b$)

$$\mathcal{L}_{\text{Yukawa}} = f_e \bar{L} H e_R + f_u \bar{Q}\tilde{H} u_R + f_d \bar{Q} H d_R + f_\nu \bar{L}\tilde{H}\nu_R (?)$$

- Gauge invariance of Yukawa terms restricts the choice of hypercharges Y down to **2** arbitrary numbers: Y_L and Y_Q

Anomaly cancellation in the Standard Model

- Triangular anomalies $U_Y(1)^3$ and $U_Y(1)SU(2)^2$ are proportional to $(Y_L + 3Y_Q)$
- Electroweak symmetry breaking leaves unbroken the electromagnetic group: $Q = T_3 + \frac{1}{2}Y$
- Anomaly-free condition means $(Q_L + 3Q_Q) = (Q_e + Q_p) = 0$

$$\boxed{\text{Standard Model fermions have non-anomalous hypercharges}}$$

$$\Updownarrow$$

$$\boxed{\text{Matter is neutral}}$$

- Current experimental bounds

$$\frac{(Q_e + Q_p)}{Q_e} < 10^{-21}$$

Anomaly cancellation in SM

- Yukawa constants in the Standard Model are very different ($f_e \sim 10^{-5} f_t$)

- It may happen that one group of chiral fermions is much heavier than the other ($m_\psi \ll m_\chi$).

- **Example:** $m_{\rm bottom} \sim 5$ GeV $\ll m_{\rm top} \sim 174$ GeV. However, SM *without* t-quark is **anomalous** – gauge invariance is broken at quantum level and the theory would lose unitarity.

- Usual logic of effective field theories tells us that contributions from heavy particles are suppressed as powers of $(E/M)^n$ ("Decoupling theorem") Appelquist, Corazzone'75

- How does anomaly cancellation works at energies $m_\psi \ll E \ll m_\chi$?

D'Hoker-Farhi terms

- Contributions from heavy particles are suppressed as powers of $g^{n_1}(E/M)^{n_2}$ Appelquist, Corazzone'75

- Chiral fermions couple to the scalar field with the Yukawa coupling constant $f \sim \frac{M}{v}$. Mass contribution can cancel no matter how high the mass is.

- Heavy **chiral** fermions can produce quantum corrections to the current, not suppressed by their mass D'Hoker-Farhi'84

$$j^{\mu}_{\text{DF}} \sim \epsilon^{\mu\nu\lambda\rho} \frac{H^* \overleftrightarrow{D}_{\nu} H}{|H|^2} F_{\lambda\rho}$$

H **– Higgs field**. This current survives even as $|H| \to \infty$

- D'Hoker-Farhi current is not conserved:

$$\partial_{\mu} j^{\mu}_{\text{DF}} \sim \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho}$$

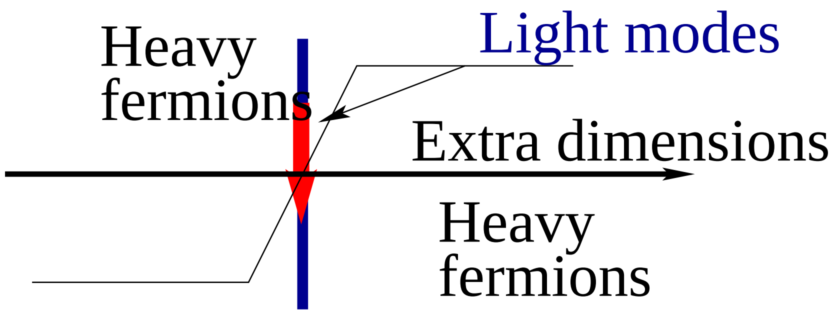
Observational signatures of anomalies

- Anomaly analysis gives information about the arbitrarily **high energy** physics
- For example, the discovery of b-quark strongly hinted at existence of the t-quark (no matter how heavy it would be)!
- Can the anomalous currents a là D'Hoker-Farhi, produced by some heavy particles, be observed at low energies?

Anomalies can probe into the high-energy physics

Example: higher-dimensional current

- Theory in 4+1 : $S = \int d^4x\, dz\, \bar{\Psi}_f(x) \Big(i\not{D} + \lambda\Phi(z) \Big) \Psi_f(x)$.



- Fermions interact with the "domain wall": $\Phi(z) = \Phi_0 \tanh(M_5 z)$ $M_5 \gg$ TeV

- Fermions in the **bulk** ($z \neq 0$) are vector-like and massive $M_\Psi = \lambda\Phi_0$.

- Zero mode in the kink background are **chiral**

Rubakov, Shaposhnikov (1983)

$$\Psi(x,z) = \psi(x) \exp\Big(\pm\lambda \int_0^z \Phi(z')dz' \Big) \qquad \boxed{\gamma_5\psi(x) = \pm\psi(x)}$$

Anomaly inflow

- Modes of only one chirality on the domain wall will produce **gauge anomaly**

What restores consistency of the theory?

- Massive bulk modes produce a **Chern-Simons** term in the effective action

$$S_{\rm CS} = \frac{\kappa}{4}\int d^4x dz\, \epsilon^{abcde} A_a F_{bc} F_{de}$$

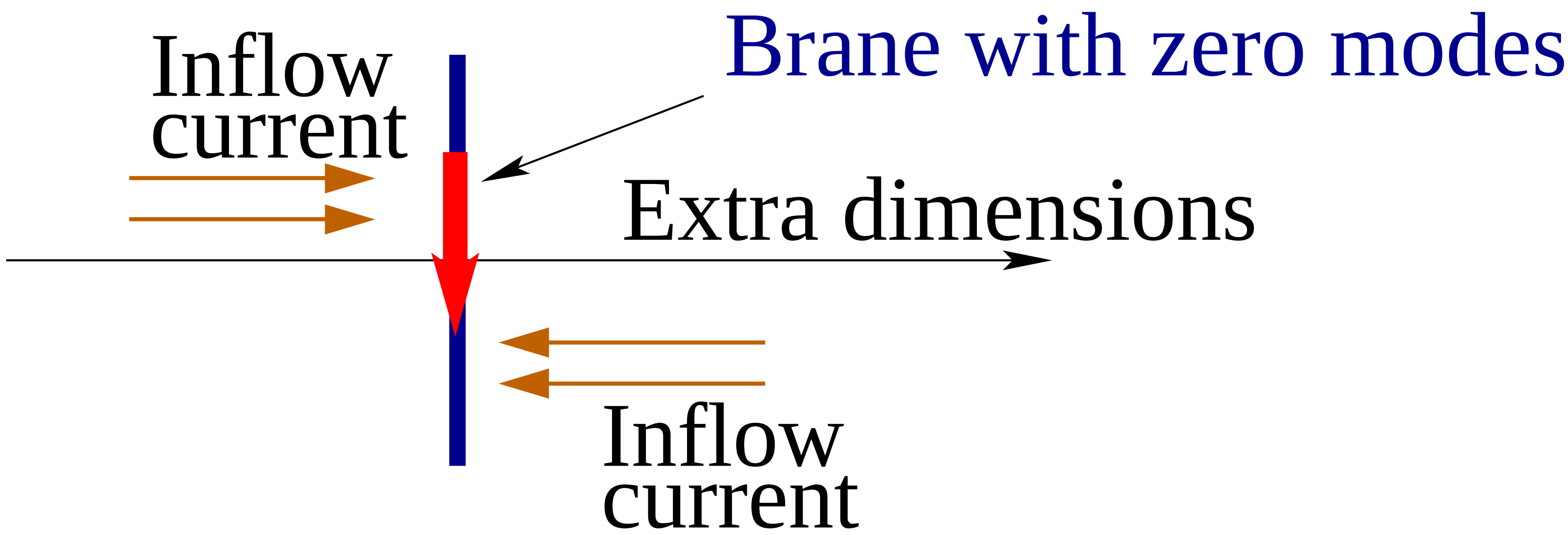
- This term leads to the current, perpendicular to the electric field:

$$J^a_{\rm CS} = \frac{\delta S_{\rm CS}}{\delta A_a} = \frac{\kappa}{4}\epsilon^{abcde} F_{bc} F_{de}$$

- If electric field is pointing along the brane, the Chern-Simons current will flow towards the brane: – **anomaly inflow** current

Faddeev, Shatashvili'84 Callan, Harvey'85

- If the extra dimension is compact (as in original Kaluza-Klein) this Chern-Simons becomes **an axion term**: $\theta \tilde{F} F$



- Chern-Simons current is conserved in the bulk, divergent on the domain wall
- Its divergence cancels the divergence of the zero mode current

$$\partial_a J^a_{\text{CS}} + \partial_\mu \langle j^\mu_{\text{zm}} \rangle = 0$$

Manifestations of Anomaly Inflow?

- Experimentally electric neutrality of matter is confirmed to a very high precision $\frac{Q_e+Q_p}{Q_e} < 10^{-21}$
- What if still $\frac{Q_e+Q_p}{Q_e} \neq 0$
- What will the 4-dimensional observer detect?
  - Flux of particles from higher dimensions?
    Wrong! Their masses are too high
  - Five-dimensional transversal photon:
    $k^\mu A_\mu + k^5 A_5 = 0$ but $k^\mu A_\mu \neq 0$?
    Wrong! – photon cannot propagate in the bulk
- The inflow current is a **vacuum** current – not carried by real particles. It is caused by a redistribution of the charges in the Dirac sea of the full theory, leads to an appearance of an electric charge on the brane.

Experimental detection of extra dimensions?

- Plane wave propagating in the strong magnetic field $H_x \approx \text{const}$ and $\kappa_0 \ll 1$. For the wave with parallel to $\vec{H}$ polarization

Boyarsky, **O.R.**, Shaposhnikov 2005; Boyarsky, **O.R** 2008

$$\frac{1}{\Delta(z)}\partial_z\Big(\Delta(z)\partial_z A_x\Big) + \Box A_x = \underbrace{\frac{\alpha_{\text{EM}}^2\kappa_0^2\vec{H}^2}{M_5^2\Delta^2(z)}}_{\text{CS term, non-perturbative in } \kappa_0!} A_x + \mathcal{O}(\kappa_0)$$

- **Massive** wave equation $\Box A_x(x) - m_{\gamma H}^2 A_x(x) = 0$
- Mass $m_{\gamma H}^2 \sim \alpha_{\text{EM}}\kappa_0|\vec{H}|$ depends only on 4-dim quantities. It is not suppressed by the scale of 5th dimension M_5
- Massless wave equation for perpendicular component
- $m_{\gamma H} \sim 3 \times 10^{-11}$ eV for $\kappa \sim 10^{-21}$
- This leads to the **ellipticity** of the linearly polarized light

$$\Delta\phi = \frac{m_{\gamma H}^2}{2\omega}L \sim \frac{\kappa_0\alpha_{\text{EM}}|\vec{H}|}{2\omega}L$$

Ellipticity in anomalous electrodynamics

- Ellipticity also appears in theories where photon interacts with ALPs, millicharged particles, etc. or due to the QED corrections to the electrodynamics Lagrangian

- Signatures of anomalous electrodynamics **differ** from these examples

- Unlike theories with ALP, here there is **no** dichroism (rotation of polarization plane) in this theory, as there are **no new light degrees of freedom**. There is also no “light shining through the wall”

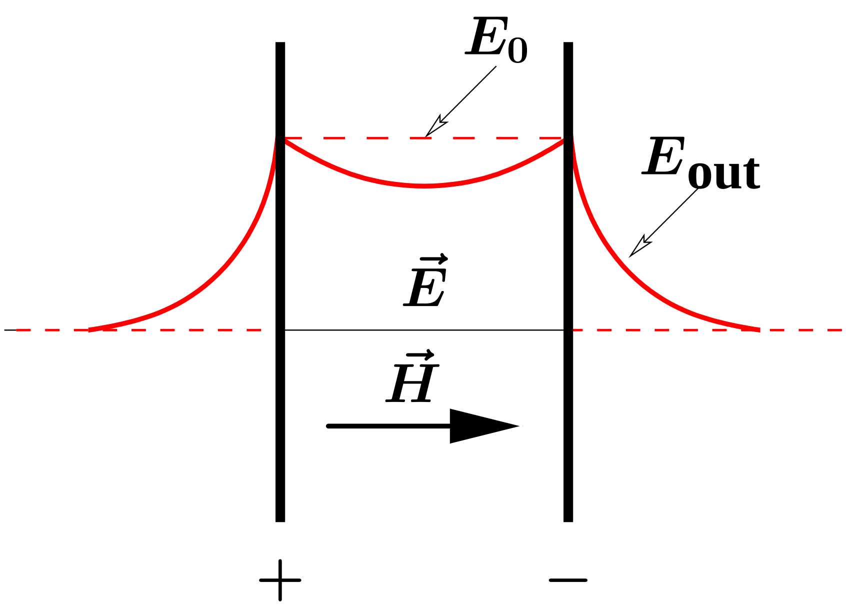
- Ellipticity in our case is proportional to the $|\vec{H}|$ (unlike QED or ALP cases, where ellipticity $\sim \vec{H}^2$). This is a **signature** of non-local (higher-dimensional) physics

Static solution in magnetic field

Static solution in strong magnetic field $\vec{H} \approx \text{const}$ and $\kappa_0 \ll 1$ to one equation for the electrostatic potential $\Phi(x,z) = \phi(x)\chi(z)$:

Boyarsky, O.R., Shaposhnikov PRD 2005

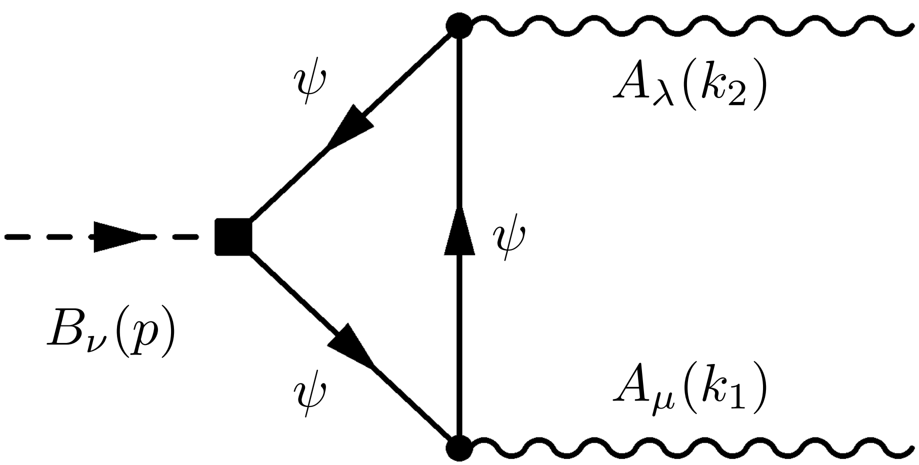
$$\frac{1}{\Delta(z)}\partial_z\Big(\Delta(z)\partial_z\Phi\Big) + \vec{\nabla}^2\Phi = \underbrace{\frac{\alpha_{\text{EM}}^2\,\kappa_0^2\,\vec{H}^2}{M_5^2\,\Delta^2(z)}}_{\text{CS term, non-perturbative in }\kappa_0}\Phi + \underbrace{e_5^2\rho(x)\delta(z)}_{\text{source charge}} + \mathcal{O}(\kappa_0)$$



- ⋆ Effective Poisson equation:
- ⋆ $\vec{\nabla}^2\phi(x) - m^2_{\gamma H}\phi(x) = \alpha_{\text{EM}}\rho(x)$
- ⋆ Electric field **gets screened** as if **photon** had **become massive**
- ⋆ Mass $m^2_{\gamma H} = \alpha_{\text{EM}}\kappa_0|\vec{H}|$ depends **only** on 4-dim quantities – fine-structure constant α_{EM} and magnetic field $\vec{H}$ as measured on the brane.

Anomalies and extra $U(1)$

- Many extensions of the SM predict theories with extra $U(1)$ fields
- SM fermions are chiral. Generically, some of these new $U(1)$ fields will have anomalous couplings



- Leads to mixed Chern-Simons term:

$$\mathcal{L}_{4d\,\mathrm{CS}} = \kappa\epsilon^{\mu\nu\lambda\rho}A_\mu X_\nu F_{\lambda\rho}$$

- The only two relevant operators (together with kinetic mixing)

Mixed anomalies

- Term $\mathcal{L}_{4d\,\mathrm{CS}} = \kappa\epsilon^{\mu\nu\lambda\rho}A_\mu X_\nu F_{\lambda\rho}$ is an exhibition of **mixed anomalies** – non-conservation of a gauge current for field X in the background of field A:

$$\partial j_X = \tilde{F}_A F_A; \qquad \partial j_A = \tilde{F}_A F_X$$

- Even if the gauge group $U(1)_A$ is vector-like (e.g. electromagnetic group $U(1)_\gamma$), anomalies $U(1)_A^2 U(1)_X$ can still be present!
- Dimension 4 operator, not suppressed at low energies, although can be generated by arbitrarily heavy (anomalously charged) fermions

Mixed anomalies involving photons

- Possibility : field A – photon, field X – new massive vector field.
- Longitudinal component behaves as derivatively coupled scalar ("Goldstone boson equivalence theorem"): $X_L = \frac{\partial\theta}{M_X}$
- Chern-Simons term is equivalent to the ALP with the mass M_X and coupling

$$\mathcal{L}_{\text{CS}} \to \frac{\kappa}{M_X}\theta\tilde{F}F$$

- Can be tested in any ALP experiments (birefringence, dichroism, shining light through the wall)
- If there is an additional light chiral fermion ψ for $E > m_\psi$ effective Lagrangian becomes **non-local**
- Now X_μ couples to the **conserved** current

Antoniadis, Boyarsky, O.R., 2008

$$J^\mu_X = \frac{\delta\mathcal{L}}{\delta X_\mu} = \kappa A \wedge F_A + \kappa\frac{\partial}{\Box}F_A \wedge F_A$$

Mixed anomalies with electroweak group

- In the Standard Model photon is a part of electroweak group
- Therefore, mixed anomaly is

$$\mathcal{L}_{\text{CS}} = \kappa \epsilon^{\mu\nu\lambda\rho} Y_\mu X_\nu (F_Y)_{\lambda\rho}$$

- Hyperfield $Y_\mu = \cos\theta_W \gamma_\mu + \sin\theta_W Z_\mu$. Any mixed anomaly leads to the massive photon?!
- Can one write terms ZXF_Z or ZXF_γ in the $U(1)_Y \times SU(2)$ invariant way?
- **Yes!** D'Hoker-Farhi type terms:

$$\kappa_1 \frac{H^\dagger D H}{|H|^2} X F_Y + \kappa_2 \frac{H F_W D H^\dagger}{|H|^2} X$$

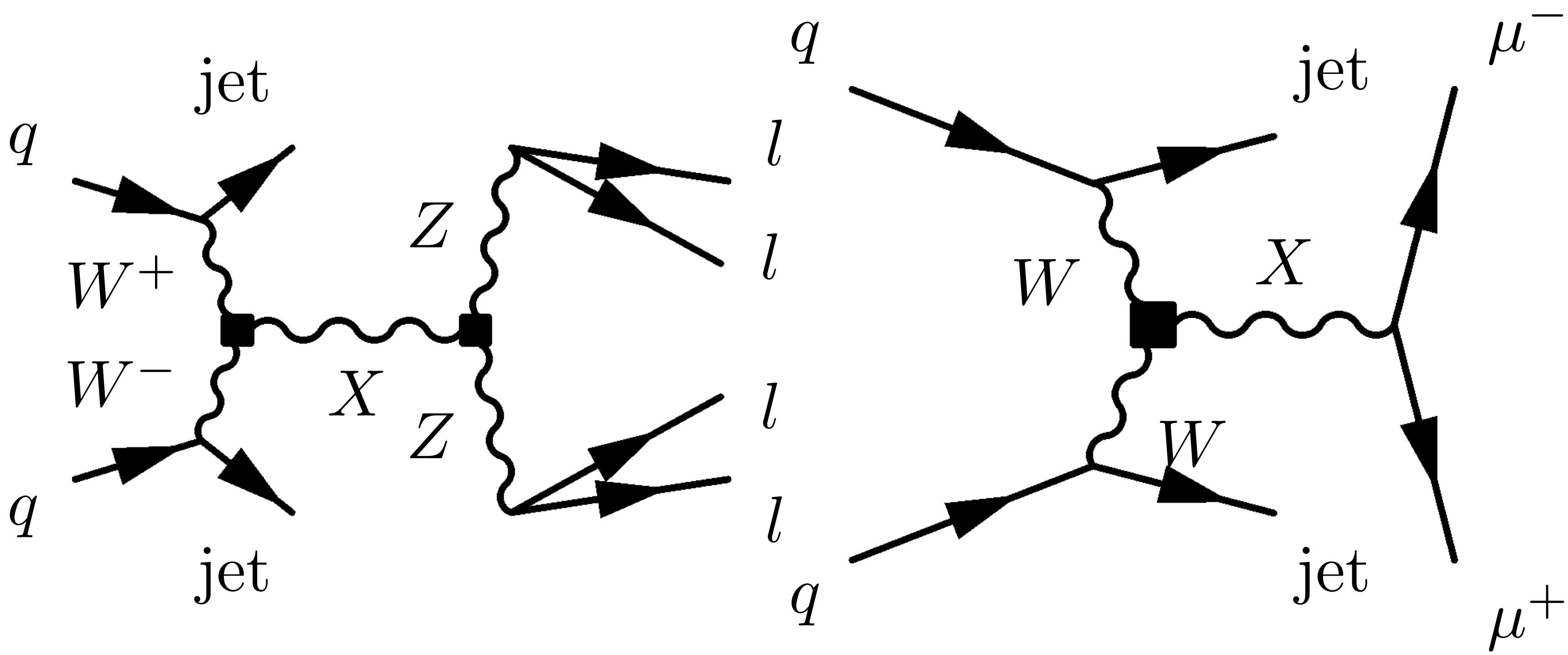
Testing anomalies at LHC

- New fermions may be **too heavy** for LHC ($M \gg 14$ TeV)
- New vector bosons may be "*light enough*" ($M_X \sim 1$ TeV) for LHC energies
- New X-boson may not interact with the SM fermions
- If new vector boson X does not couple to the SM fields – no chance to see it?!
- Triangular diagrams with heavy fields may lead to anomalous couplings of X boson with with vector bosons $W^{\pm}, Z$:

$$\kappa\epsilon^{\mu\nu\lambda\rho}X_\mu Z_\nu \partial_\lambda Z_\rho, \quad \kappa\epsilon^{\mu\nu\lambda\rho}X_\mu W^+_\nu \partial_\lambda W^-_\rho$$

- Its mixed anomaly with the SM fields may give interesting observable effects.

LHC signatures



- For $\kappa \sim 1$, mass $M_X \sim 1$ TeV and **arbitrarily** heavy new fields these theories are **testable** at LHC

Conclusion

- Theories with non-trivial anomaly cancellation are similar to ALPs
- Some of anomaly motivated models do not introduce new light particles, but still predict **effects in strong magnetic field**
- They may behave differently at higher energies, evading astrophysical bounds
- Experiments (such as PVLAS-II, ALPS, OSQAR, . . .) can also probe for the signatures of these theories (and e.g. **discover extra dimensions**!)
- There is an alternative approach to probe these theories – **static** experiments measuring modifications of Coulomb law in the magnetic field
- Anomalies open a possibility to see (otherwise hidden) new vector bosons at LHC

Thank you for your attention!

The End

Anomalous Extensions of SM

- Choice of hypercharges in SM is controlled by Yukawa interaction.
- This fixes hypercharge assignments up to two constants: κ_l in lepton sector and κ_q in quark sector.
-

e_L	e_R	ν_L	Q_L	u_R	d_R	ν_R
$-1+\kappa_l$	$-2+\kappa_l$	$-1+\kappa_l$	$\frac{1}{3}+\kappa_q$	$\frac{4}{3}+\kappa_q$	$-\frac{2}{3}+\kappa_q$	κ_l

- These constants are usually chosen to be zero to ensure that SM is anomaly free:

$$\partial_\mu j_Y^\mu = \frac{\mathrm{Tr}[Y^3]}{16\pi^2}\epsilon_{\mu\nu\lambda\rho}F_Y^{\mu\nu}F_Y^{\lambda\rho} + \frac{\mathrm{Tr}[Y_L]}{16\pi^2}\epsilon^{\mu\nu\lambda\rho}\,\mathrm{Tr}_{SU(2)}\,G_{\mu\nu}G_{\lambda\rho}$$

where $\mathrm{Tr}[Y^3] = 6(\kappa_l + 3\kappa_q)$ and $\mathrm{Tr}[Y_L] = -2(\kappa_l + 3\kappa_q)$

- Experimentally $\kappa_l + 3\kappa_q = \frac{e-p}{e} < 10^{-21}$
- This number is small but may be **non-zero** if SM is a sector a bigger theory. For example, a theory with extra dimensions

Vector-like Electrodynamics

- Arbitrary choice of parameters κ_l, κ_q leads to anomaly of **hypercharge current**.
- However, for **any** choice of hypercharges, electrodynamics remains vector-like and **anomaly-free**
- If SM is expanded by some additional **4-dim fields**, it may happen that the electrodynamics will also become chiral
- If the theory contains additional U(1) **4-dim fields**, there can **mixed anomaly**. These **anomalies** of SM can be canceled by inflow from extra dimensions
- What are the consequences of the presence of inflow currents from the point of view of the low energy physics on a brane?

Anomalous Electrodynamics

Consider again our simplest example of anomalous electrodynamics on a domain wall in 4+1 dimensions (z – **coordinate of 5th dim**)

$$S = \underbrace{-\frac{1}{4e_5^2}\int \Delta(z) F\wedge\star F}_{\text{5-dim kinetic term}} + \underbrace{\frac{1}{4}\int \kappa(z) A\wedge F\wedge F}_{\text{Anomaly inflow interaction}} + \underbrace{\int d^4x\, \mathcal{L}_{matter}}_{\text{Anomalous theory: } \partial_\mu j^\mu \sim F\tilde{F}}$$

Oda 2000; Dubovsky et al. 2000; Shaposhnikov, Tinyakov 2001

Factor $\Delta(z) = \exp(-2M|z|)$ is responsible for localization of the gauge fields on a brane

Without CS this action would describe a 4-dim theory for $E < M$

Normalizable zero mode of gauge fields: $\partial_z F_{\mu\nu} = 0, \quad F_{\mu z} = 0$

Equations of motion

Set of **non-linear** 5-dimensional Maxwell-like equations:

$$\partial_b\Big(\Delta(z)F^{\mu b}\Big) = e_5^2\Big(J^\mu_{\text{CS}} + j^\mu_{\text{SM}}\Big) \qquad a,b = 0,\ldots,4$$

$$\Delta(z)\partial_\mu F^{z\mu} = e_5^2 J^z_{\text{CS}} \qquad \mu = 0,\ldots,3.$$

$$J^\mu_{\text{CS}} = 3\kappa(z)\epsilon^{\mu\nu\lambda\rho}F_{z\nu}F_{\lambda\rho}$$

$$J^z_{\text{CS}} = \frac{3}{4}\kappa(z)\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$$

Inflow current J_{CS} **cancels anomaly on the brane:**

$$\partial_\mu J^\mu_{\text{CS}} + \partial_z J^z_{\text{CS}} + \partial_\mu j^\mu_{\text{SM}} = 0$$

Plane wave propagating in the strong magnetic field $H_x \approx \mathrm{const}$ and $\kappa_0 \ll 1$. For the wave with parallel to $\vec{H}$ polarization

Boyarsky, O.R 2007

$$\frac{1}{\Delta(z)}\partial_z\Big(\Delta(z)\partial_z A_x\Big) + \Box A_x = \underbrace{\frac{\alpha_{\text{EM}}^2\kappa_0^2\vec{H}^2}{M_5^2\Delta^2(z)}}_{\text{CS term, non-perturbative in } \kappa_0!}A_x + \mathcal{O}(\kappa_0)$$

- ⋆ **Massive** wave equation $\Box A_x(x) - m_{\gamma H}^2\, A_x(x) = 0$
- ⋆ Mass $m_{\gamma H}^2 \sim \alpha_{\text{EM}}\kappa_0|\vec{H}|$ depends only on 4-dim quantities. It is not suppressed by the scale of 5th dimension M_5
- ⋆ Massless wave equation for perpendicular to the magnetic field component $\Box A_y(x) = 0$
- ⋆ This leads to the **ellipticity** (birefringence) of the linearly polarized light $\Delta\phi = \frac{m_{\gamma H}^2}{2\omega}L \sim \frac{\kappa_0\alpha_{\text{EM}}|\vec{H}|}{2\omega}L$

Anomalies in SM on D-branes

- Appearance of additional **anomalous U(1) groups** is a generic feature in D-brane constructions of SM

Ibanez, Rabadan, Uranga'98

- Anomalous parameter can have **arbitrary** values

Antoniadis, Kiritsis, Rizos'02

- Effects, similar to those, appearing in SM can be induced via anomalous $\gamma\gamma\gamma'$ **coupling**.

Antoniadis, Boyarsky, O.R in progress

- This may produce the low-energy string theory signature not suppressed by string scale M_s?!

Conclusion

- In theories with **anomaly inflow** the electric charge, placed in a magnetic field, gets screened. This **low-energy** effect can serve as a **signature of extra dimensions.**
- Modern experimental data shows that our world is non-anomalous with a very high precision. However, with these restrictions in mind **the effect can be pronounced enough to be detected.**
- Any higher-dimensional theory should either present a mechanism ensuring that the brane world is non-anomalous or explain a **fine-tuning** of the hypercharges.
- Anomalous U(1) couplings generically appear in string vacua. Possible **experimental tests of string theory**?

STATIC SOLUTION

Five coupled non-linear equations reduce for $\vec{H} \approx \mathrm{const}$ and $\kappa_0 \ll 1$ to one equation for the electrostatic potential $\Phi(x, z) = \phi(x)\chi(z)$:

$$\partial_z\Big(\Delta(z)\partial_z\Phi\Big)+\Delta(z)\vec{\nabla}^2\Phi = \underbrace{\frac{\alpha_{\mathsf{EM}}^2\kappa_0^2\vec{H}^2}{M_5^2\Delta(z)}\Phi}_{\text{CS current}} + \underbrace{e_5^2 q(x)\delta(z)}_{\text{source charge}} + \mathcal{O}(\kappa_0)$$

non-perturbative in κ_0!

$$\chi(z) \approx \exp\left(-\frac{m_{\gamma H}^2}{M_5^2}e^{2M_5|z|}\right) \qquad \vec{\nabla}^2\phi(x) - m_{\gamma H}^2(x) = \alpha_{\mathsf{EM}} q(x)$$

$$m_{\gamma H}^2 = \alpha_{\mathsf{EM}}\kappa_0|\vec{H}|$$

Go back...

Bulk theory

A model of localization of both fermions and gauge fields.

$$S = -\frac{1}{4e_5^2}\int d^5x\,\Delta(z)F_{ab}^2 + \int d^5x\,\sum_{f=1}^{2}\bar{\Psi}_f(x)\Big(i\not{D}_f + m_f(z)\Big)\Psi_f(x).$$

There are two fermions $\Psi_{1,2}$, interacting with the gauge field with the different charges: $\not{D}_f = \not{\partial} + \frac{e_f}{e_5}\not{A}$, $e_1 \neq e_2$. The fermionic mass terms $m_1(z) = -m_2(z)$ have a “kink-like” structure in the direction z: $m_1(z \to \pm\infty) \to \pm m_\psi$.

MASSES FOR FERMIONIC ZERO MODES

The only way to make the electro-dynamics anomalous is to take left and right moving fermions with different electric charges. Thus one can only introduce a mass term via the Higgs mechanism with an electrically charged Higgs field:

$$S_\phi = \int d^5x \left[\left|D_a\phi\right|^2 - m_\phi^2(z)|\phi|^2 - \frac{\lambda}{4}|\phi|^4 + f\bar{\Psi}_1\Psi_2\phi + \mathrm{h.c.} \right] ,$$

where $D_\mu\phi = i\partial_\mu\phi + (\frac{e_L}{e} - \frac{e_R}{e})A_\mu\phi$ and the Higgs mass $m_\phi^2(z)$ is negative at $z = 0$ and tends to the positive constant in the bulk, as $|z| \to \infty$.

EFFECTIVE FIELD THEORIES AND "DECOUPLING THEOREM"

The usual logic behind **effective field theories**: integration of massive fields only leads to renormalization of charges and fields, while all additional interaction suppressed by some positive power of E/M.**[“Decoupling theorem” Appelquist,Corazzone’75]**

Question: if the mass scale of extra dimensions is much bigger than our present energies — can one still expect to see any low energy signatures?

Yes! The “decoupling theorem” does not always hold. The most famous counterexample: theories, with **Chern-Simons-like** interactions.**[Redlich’83]**

- In 2+1 dimensions:

$$\log\det(i\gamma^\mu\partial_\mu + M + e\gamma^\mu A_\mu) = \frac{e^2}{8\pi^2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda + \dots$$

- Chern-Simons term survives even as $M \to \infty$!
- True in any **odd** space-time dimensions.
- **What about 3+1 dimensions?**

$U(1)^3$ AND $U(1) \times SU(2)^2$ ANOMALY

$$U(1)^3 \ : \ \partial_\mu j_Y^\mu = \frac{\mathrm{Tr}[Y^3]}{16\pi^2}\epsilon_{\mu\nu\lambda\rho}F_Y^{\mu\nu}F_Y^{\lambda\rho} + \frac{\mathrm{Tr}[Y_L]}{16\pi^2}\epsilon^{\mu\nu\lambda\rho}\,\mathrm{Tr}_{SU(2)}\,G_{\mu\nu}G_{\lambda\rho}$$

$$U(1) \times SU(2)^2 \ : \ D^\mu j_\mu^\alpha = \frac{\mathrm{Tr}[Y_L]}{8\pi^2}\epsilon^{\mu\nu\lambda\rho}G^\alpha_{\mu\nu}F_{\lambda\rho}$$

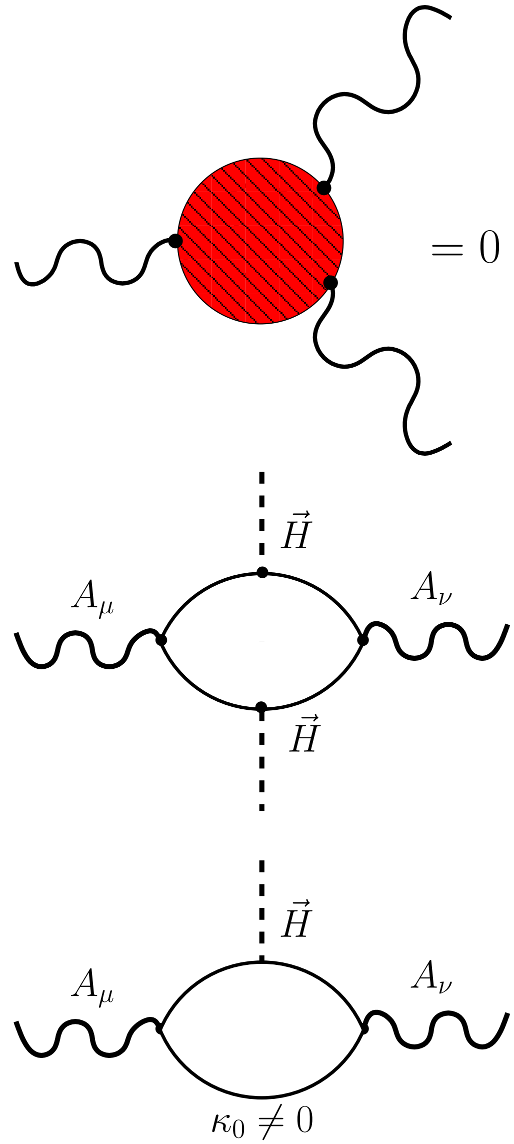
Signatures of Extra Dimensions

- New particles ("KK towers") appear. SM particles disappear into bulk. **High-energy signatures:** only at energies above the **mass gap**.

- Certain theories lead to a modification of Newtons's law at sub-mm scales – **low-energy signature** **[Arkani-Hamed,Dimopoulos,Dvali'98]**

- Theories with **anomaly inflow:** special type of brane-bulk interaction, **not suppressed** by a mass gap. Low-energy signatures? **[This talk]**

- **Furry theorem in QED:** any diagram with odd number of external photon legs is zero (CP-symmetry).
- **QED** corrections to Maxwell theory
  $\mathcal{L} = -\frac{1}{4}F^2_{\mu\nu} + \frac{14\,\alpha^2_{\text{EM}}}{45\,m_e^4}(\vec{E}\vec{H})^2 + \ldots$
- The Euler-Heisenberg Lagrangian gives ellipticity but does not lead to the photon mass. Static (capacitor) experiment would give no results
- Once $\kappa_0 \neq 0$ there is no **Furry theorem** as $e_L \neq e_R$. Anomalous triangular diagram exists and leads to a pole in the photon propagator with $m^2_{\gamma H} \sim \kappa_0 \alpha_{\text{EM}} |\vec{H}|$



There is also another experimental setup, which can observe anomaly inflow and distinguish 5-dimensional theory from its 4-dimensional counterparts

ANOMALOUS $\gamma\gamma Z$ COUPLING

In SM there can be only $\gamma\gamma Z$ anomalies. ***The analysis gets messy***

$$\partial_\mu j_Z^\mu = -\frac{4N_f\,(\kappa_l + 3\kappa_q)}{\pi^2 \sin 2\theta_W}\vec{E}_\gamma\cdot\vec{H}_\gamma\ ;\ \partial_\mu j_\gamma^\mu = -\frac{8N_f(\kappa_l + 3\kappa_q)}{\pi^2 \sin 2\theta_W}(\vec{E}_\gamma\cdot\vec{H}_Z+\vec{E}_Z\cdot\vec{H}_\gamma)$$

- A background (capacitor) with $\vec{E}\cdot\vec{H} \neq 0$ creates an **inflow of Z current**
- Anomalous density of Z charge creates Z field and **non-trivial γZ background**
- Non-trivial γZ background leads to **inflow of electro-magnetic current**
- Anomalous distribution of electric charge on the brane is created and electric field is modified **as if photon has acquired mass**

$$m^2_{\gamma H} = \frac{2N_f e_4^2|\vec{H}|}{\pi^2 \sin 2\theta_W}(\kappa_l + 3\kappa_q)$$

does not depend on m_Z or M_5!

SETUP OF STATIC EXPERIMENT

- In the SM $\kappa \lesssim 10^{-21}$ which leads to the $m_{\gamma H} \lesssim 10^{-10}$ **eV** for the magnetic field 10 Tesla.
- **Idea N° 1:** if one “turns on” mass for the photon, the capacitance of a system would change $\Rightarrow$ Create an RC-circuit, turn on strong magnetic field and measure the shift of capacitance. The change of capacitance $\frac{\Delta C}{C} \sim m_{\gamma H}$. Possible to measure shift of capacitance with **femto**Farad (10^{-3}pF) precision and thus masses $m_{\gamma H} \gtrsim 10^{-8}$ eV
- **Idea N° 2:** Attraction force between two charged parallel plates (ideal capacitor) can be measured with **nano**Newton precision. Can probe mass range $m_{\gamma H} \gtrsim 10^{-11}$ eV.
- Tentative limit on measurements of deviation from the Gauss law $\sim 10^{-14}$ eV
- Unique signature $m_{\gamma H} \sim \sqrt{|\vec{H}|}$

Equations for γ field

$$\begin{cases} \partial_z\Big(\Delta(z)\partial_z\Phi_\gamma\Big) + \Delta(z)\nabla^2\Phi_\gamma = -e_5^2\Big(q(x)\delta(z) + j^0_{\mathsf{DF}} + J^0_{\mathsf{CS},\gamma}\Big)\,, \\ \partial_z\Big(\Delta(z)F^{xz}\Big) + \dfrac{\Delta(z)}{r}\partial_r\Big(rF^{xr}\Big) = e_5^2\Big(j^x_{\mathsf{DF}} + J^x_{\mathsf{CS},\gamma}\Big)\,, \\ \partial_z\Big(\Delta(z)F^{rz}\Big) + \Delta(z)\partial_x F^{rx} = e_5^2\Big(j^r_{\mathsf{DF}} + J^r_{\mathsf{CS},\gamma}\Big)\,, \\ \partial_z\Big(\Delta(z)F^{\theta z}\Big) + \Delta(z)\partial_x F^{\theta x} + \dfrac{\Delta(z)}{r}\partial_r\Big(rF^{\theta r}\Big) = e_5^2\Big(j^\theta_{\mathsf{DF}} + J^\theta_{\mathsf{CS},\gamma}\Big)\,, \\ \Delta(z)\Big(\partial_x F^{xz} + \dfrac{1}{r}\partial_r\left(rF^{rz}\right)\Big) = -e_5^2 J^z_{\mathsf{CS},\gamma}\,, \end{cases}$$

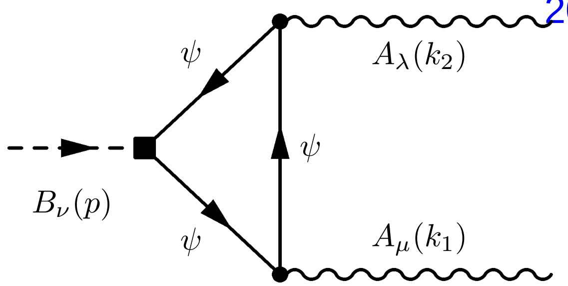
Equations for Z field

$$\begin{cases} \partial_z\Big(\Delta(z)\Phi_{\mathsf{Z}}\Big) + \Delta(z)\nabla^2\Phi_{\mathsf{Z}} - e_5^2 m_{\mathsf{Z}}^2(z)\Phi_{\mathsf{Z}} = -e_5^2\Big(q_{\mathsf{Z}}(x)\delta(z) + j^0_{\mathsf{DF,Z}} + \mathcal{J}^0_{\mathsf{CS,Z}}\Big)\,, \\ \partial_z\Big(\Delta(z)\mathcal{F}^{xz}\Big) + \dfrac{\Delta(z)}{r}\partial_r\Big(r\mathcal{F}^{xr}\Big) - e_5^2 m_{\mathsf{Z}}^2(z)\mathcal{A}^x = e_5^2\Big(j^x_{\mathsf{DF,Z}} + \mathcal{J}^x_{\mathsf{CS,Z}}\Big)\,, \\ \partial_z\Big(\Delta(z)\mathcal{F}^{rz}\Big) + \Delta(z)\partial_x\mathcal{F}^{rx} - e_5^2 m_{\mathsf{Z}}^2(z)\mathcal{A}^r = e_5^2\Big(j^r_{\mathsf{DF,Z}} + \mathcal{J}^r_{\mathsf{CS,Z}}\Big)\,, \\ \partial_z\Big(\Delta(z)\mathcal{F}^{\theta z}\Big) + \Delta(z)\partial_x\mathcal{F}^{\theta x} + \dfrac{\Delta(z)}{r}\partial_r\Big(r\mathcal{F}^{\theta r}\Big) - e_5^2 m_{\mathsf{Z}}^2(z)\mathcal{A}^\theta = e_5^2\Big(j^\theta_{\mathsf{DF,Z}} + \mathcal{J}^\theta_{\mathsf{CS,Z}}\Big)\,, \\ \Delta(z)\Big(\partial_x\mathcal{F}^{xz} + \dfrac{1}{r}\partial_r\left(r\mathcal{F}^{rz}\right)\Big) + e_5^2 m_{\mathsf{Z}}^2(z)\mathcal{A}^z = -e_5^2\mathcal{J}^z_{\mathsf{CS,Z}}\,. \end{cases}$$

back to $\gamma\gamma Z$

EXAMPLE 2: NEW VECTOR FIELD AND CS TERMS

- In the SM model fermions have both vector and axial couplings to gauge fields (e.g. $e^{\pm}$ interaction with electromagnetic and Z field)
- Imagine an extension of the SM where some fermions (either SM or new ones) interact with both photon A_μ and new gauge field B_μ Antoniadis, Boyarsky, O.R 2006
- Anomalous triangular diagram induces **4-dim** Chern-Simons-like coupling between two fields:



$$\mathcal{L}_{\text{CS}} = \kappa\epsilon^{\mu\nu\lambda\rho}A_\mu B_\nu \partial_\lambda A_\rho$$

- We obtain an effective theory

$$\mathcal{L} = -\frac{1}{4}|F_A|^2 - \frac{1}{4}|F_B|^2 + \frac{m_B^2}{2}|d\theta + B|^2 + \kappa A\wedge B\wedge F_A + \kappa\theta F_A\wedge F_A$$

LONGITUDINAL COMPONENT AT LOW ENERGIES

$$\mathcal{L} = -\frac{1}{4}|F_A|^2 - \frac{1}{4}|F_B|^2 + \frac{m_B^2}{2}|d\theta + B|^2 + \kappa A \wedge B \wedge F_A + \kappa\theta F_A \wedge F_A$$

- The theory is gauge invariant with respect to variation of the $B_\mu \to B_\mu + \partial_\mu\lambda$ and $\theta = \theta - \lambda$.
- However, B_μ couples to the current which is **not conserved**:

$$J_B^\mu = \frac{\delta\mathcal{L}}{\delta B_\mu} = \kappa A \wedge F_A; \qquad \partial_\mu J_B^\mu = \kappa F_A \wedge F_A$$

- Longitudinal component of the B-field **does not decouple** and **behaves as ALP** with mass m_B and coupling $M_{\text{ALP}} = \frac{m_B}{\kappa}$

LONGITUDINAL COMPONENT AT HIGH ENERGIES

- If there is an additional massive particle (with **mass m_0**), interacting with A_μ and B_μ, for $E > m_0$ effective Lagrangian becomes non-local

- Now B_μ couples to the **conserved** current

Antoniadis, Boyarsky, O.R., to appear

$$J_B^\mu = \frac{\delta \mathcal{L}}{\delta B_\mu} = \kappa A \wedge F_A + \kappa \frac{\partial}{\Box} F_A \wedge F_A$$

- For example, fermions with mass m_0 will produce the following term in the effective action

$$\mathcal{L}_\psi = \kappa \left(\theta \frac{m_0^2}{\Box + m_0^2} - \partial_\mu B^\mu \frac{1}{\Box + m_0^2} \right) F_A \wedge F_A$$

- At energies $E \gtrsim m_0$ the production of the longitudinal polarization is suppressed as $(m_B/E)^2$

- If 1 eV $< m_0 <$ 1 keV we will have effects in laboratory but not in stars!

EXPERIMENTAL DETECTION?

Model	κ_0	$m_{\gamma H}$, GeV	τ_0, sec	L_0, cm	E_{out}/E_0
new generation	1	10^{-8}	3×10^{-16}	10^{-5}	0
charged ν	10^{-15}	4×10^{-16}	10^{-8}	3×10^{2}	~ 1
electric neutrality	10^{-21}	4×10^{-19}	10^{-5}	3×10^{5}	$\sim 10^{-3}$
massive γ	10^{-36}	10^{-26}	3×10^{2}	10^{13}	$\sim 10^{-10}$

$\tau_0 \sim 1/m_{\gamma H}$ — characteristic time over which the electric field reaches its final state.

E_{out} — the value of the electric field outside the plates of a capacitor at distances much smaller than $L_0 \sim 1/m_{\gamma H}$.

An initial value of electric field $E_0 \sim 10^7$ Volt/m, magnetic field $H \sim 10^5$ Gauss, the distance between the plates $d = 10^2$ cm.

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